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Analysis of GPS-based Real Time Attitude Determination System for ITS Application

Final Report

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16. Abstract (Limit: 250 words)

This work describes the development and testing of GPS-based attitude and heading determination system (AHRS) using single-frequency (L1) carrier phase differential GPS (CPDGPS). Vehicle's attitude can be uniquely determined from two non-collinear relative position vectors, known as the baseline vectors. The accuracy of the resulting attitude estimate depends on the accuracy of the baseline vector estimates and their respective magnitudes (length). The shorter the baseline, the higher the vector accuracy required to give the same attitude accuracy that can be obtained through longer baseline system. Issues such as ambiguity resolution and phase center variations are discussed. Test result shows that single-frequency CPDGPS is still a challenge, mainly caused by the integer ambiguity problem inherent to CPDGPS problem. A more feasible but less accurate method using a short baseline is also discussed. Phase center calibration remains a challenge for this attitude determination system.

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EXECUTIVE SUMMARY

This work describes the development and testing of GPS-based attitude and heading determination system (AHRS) using single-frequency (L1) carrier phase differential GPS (CPDGPS). Vehicle's attitude can be uniquely determined from two non-collinear relative position vectors, known as the baseline vectors. The accuracy of the resulting attitude estimate is proportional to the accuracy of the baseline vector estimates and inversely proportional to their respective magnitudes (length). The feasibility of devising a real-time GPS-based AHRS that suits ITS applications are discussed. Integer ambiguity resolution techniques such as LAMBDA and attitude search method were investigated and test results reveal some challenges associated with LAMBDA encountered in real-time operation. Attitude search is found to have superior performance and is recommended for devising a GPS-based AHRS. Phase center variation error is discussed and lessons learned from the preliminary test conducted are presented.

CHAPTER 1

INTRODUCTION

1.1. ATTITUDE DETERMINATION PROBLEM

Attitude is a term used to describe the orientation of a platform with respect to a reference coordinate frame. More precisely attitude is an ensemble of parameters that describes a rotation sequence. This rotation sequence transforms a coordinate frame that is initially aligned to the reference frame to a frame that is parallel to a frame fixed to the vehicle and rotates together with it. For most vehicle navigation and guidance purposes, this reference frame is the local North-East-Down (NED) coordinate, and the vehicle carried frame is usually called the body frame. There are several ways to represent attitude, some of the most popular forms are direction cosine matrix (DCM), Euler angles[1], and quaternions[2]. Despite being equivalent, each entails different mathematical properties and representation. In this work, Euler angle representation is chosen due to its intuitive nature. A complete set of Euler angles consists of three rotation angles a sequence that describes the order of rotation. We adopt the aerospace rotation sequence, which is also known as the 3-2-1 rotation sequence[3]. This sequence defines the platform's roll (ϕ), pitch (θ), and yaw (ψ) angles (see Figure 1.1). Yaw angle is also known as the heading angle because it gives the orientation of the vehicle with respect to true North.

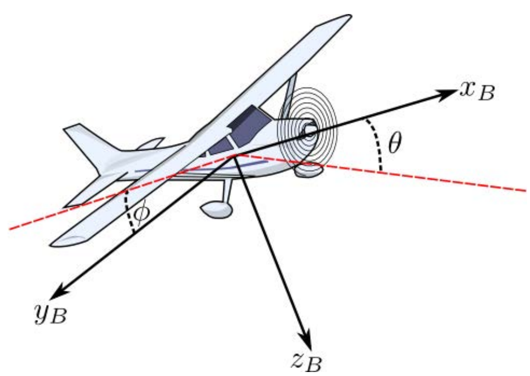

Figure 1.1: Pitch and roll angles

Since attitude measures rotation, it is naturally measured using a rotation sensor, namely the gyroscope (or gyro in short). With the advent of MEMS technology, nowadays a wide selection of gyros is available off-the-shelves with quality ranging from the expensive navigation grade rate integrating gyros or rate gyros to a wide selection of cheap automotive grade rate gyros[4]. As shown in [5], when operated on stand-alone mode, attitude obtained by purely integrating rate gyro output will have unbounded error growth. The speed at which this error grows largely depends on the sensor quality. Navigation grade gyro typically has error in the order of 0.1° after 1 hour of navigation. On the other hand, an automotive grade gyro can have error in the order a couple degrees after several minutes of navigation. Therefore, unless the gyro is of tactical grade or higher, it is necessary to fuse the attitude obtained from integrating the rate gyro with an aiding system that has long term stability in its error characteristics. To this day, global navigation satellite system (GNSS) remains the most popular aiding system to bound the error growth of the solution obtained from purely integrating inertial sensors output forward in time.

Global positioning system (GPS) is the GNSS maintained by the United States' Department of Defense. Part of the system is a set of satellite constellation which broadcast radio frequency (RF) signal that can be used to determine the position of a receiver (in this case the antenna of a receiver) anywhere on the earth that can receive the signal. GPS positioning works based on multilateration method using the range measurements obtained from the satellite's broadcast [6]. There are two kind of ranging signals that can be obtained from the broadcast signal. The first signal is known as the C/A signal, which stands for coarse acquisition signal. This is the standard ranging signals that is used in all GPS receivers to determine one's location. The tracking accuracy of this signal is in the order of 1-3 meters which translates to several meters of position error. The other signal is called the carrier phase signal. This signal can be measured with accuracy down to the millimeter level which allows it to have superior position accuracy compared to the standard position solution computed using C/A signal. The carrier phase measurement describes the distance between the antenna and the satellite in terms of the phase of the sinusoidal signal being tracked. Since the phase of any cycle is not different from any other cycles, unlike the C/A signal, this measurement is ambiguous in nature. The process of resolving this ambiguity is known as ambiguity resolution. Since this ambiguity always represents a whole number of cycle (i.e. an integer) that cannot be detected after signal acquisition and tracking, any resolution techniques must take into account the integer nature of the ambiguity being solved; hence the name integer ambiguity resolution [7].

In short, for each differential position vector $\mathbf{b} = [b_N \quad b_E \quad b_D]^T$, the double-differenced carrier phase measurements can be written as follows

$$\nabla\Delta\phi = \mathbf{H} \cdot \mathbf{b} + \lambda \cdot \nabla\Delta\mathbf{N} + \epsilon \tag{1.1}$$

where

$$\mathbf{H} = \begin{bmatrix} & \vdots & \\ \mathrm{LOS}_N^i & \mathrm{LOS}_E^i & \mathrm{LOS}_D^i \\ & \vdots & \end{bmatrix} \tag{1.2}$$

is the geometry matrix built by stacking the transpose of the line-of-sight (LOS) vector to the i-th satellite [6]. Each row corresponds to the i-th entry in the double-differenced carrier phase measurements $\nabla\Delta\phi$, and the i-th entry in the double-differenced integer ambiguity vector $\nabla\Delta\mathbf{N}$. ϵ represents other residual errors that remain after double differencing the carrier phase measurement which include the thermal noise and double differenced PCV error. Once the integer ambiguities are solved for each satellite, the baseline vector can be found by using the following weighted least square solution:

$$\mathbf{b} = \left(\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{R}^{-1}\left[\nabla\Delta\phi - \lambda \cdot \nabla\Delta\mathbf{N}\right] \tag{1.3}$$

$\mathbf{R}$ is the matrix of measurement noise covariance, $\mathbb{E}\left(\epsilon\epsilon^T\right)$, where $\mathbb{E}(\cdot)$ denotes the expectation operator. For more details on this measurement model and the double difference technique for differential GPS, we refer to literature such as [6, 7, 8].

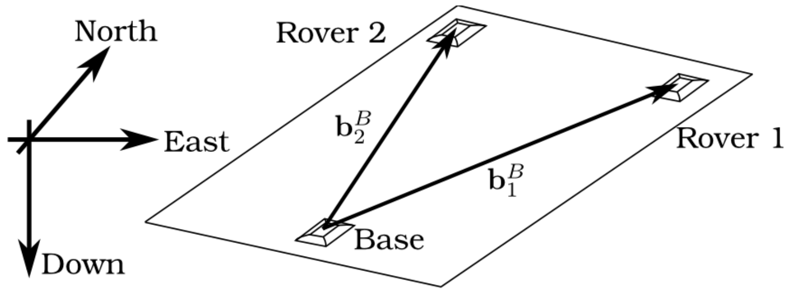

Figure 1.2: Baseline vectors

Attitude determination by using position vector falls under the class of problem known as Wahba's problem [9]. Attitude can be uniquely determined by using observation of two non-colinear vectors, known as the baseline vectors, in NED frame. They can be obtained from differential GPS solution, namely from a "base" antenna to two other antennas called "rover 1" and "rover 2" (see Figure 1.2). With the differential position obtained in the NED frame, the following equation can be formed:

$$\begin{bmatrix}\mathbf{b}_1^B & \mathbf{b}_2^B & \mathbf{b}_1^B \times \mathbf{b}_2^B\end{bmatrix} = C_N^B \begin{bmatrix}\mathbf{b}_1^N & \mathbf{b}_2^N & \mathbf{b}_1^N \times \mathbf{b}_2^N\end{bmatrix} \tag{1.4}$$

where $\mathbf{b}_k^B$ for $k = 1, 2$ indicates the i-th baseline vector resolved in the body frame, and $\mathbf{b}_k^N$ refers to the vector resolved in the NED frame. $\mathbf{b}_k^B$ is known with good accuracy since they can be pre-surveyed and measured carefully. $\mathbf{b}_k^N$ can be obtained from the positioning system. The 3×3 matrix C_N^B is the sought rotation matrix that transforms vectors resolved in the NED frame to the body frame. This matrix is a function of the vehicle's attitude (roll ϕ, pitch θ and yaw ψ) as shown in Equation (1.6). Because $\mathbf{b}_1$ and $\mathbf{b}_2$ are not colinear, the matrices are invertible and thus, C_N^B can be obtained from Equation (1.4), i.e.

$$C_N^B = \begin{bmatrix}\mathbf{b}_1^B & \mathbf{b}_2^B & \mathbf{b}_1^B \times \mathbf{b}_2^B\end{bmatrix} \begin{bmatrix}\mathbf{b}_1^N & \mathbf{b}_2^N & \mathbf{b}_1^N \times \mathbf{b}_2^N\end{bmatrix}^{-1} \tag{1.5}$$

$$C_N^B = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\sin\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\sin\theta \end{bmatrix} \tag{1.6}$$

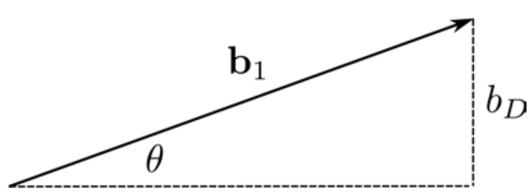

Figure 1.3: 1-D pitch angle determination from a single baseline vector

The accuracy of the attitude solution obtained through position vectors is directly a function of the position accuracy. For the same baseline vector length, the more accurate the position vector we have, the more accurate the attitude solution is. To illustrate this, we take the example of pitch angle determination from a single position vector (see Figure 1.3). The pitch angle error $\delta\theta$ is related to the error in the down vector component δb_D through the following relationship:

$$\delta\theta \approx \frac{\delta b_D}{||\mathbf{b}_1||} \tag{1.7}$$

This shows that for a 1 m baseline, 10 cm accuracy in the position vector leads to 6 degrees of error in the attitude. Another way to increase the accuracy of the attitude solution is by increasing the baseline vector length $||\mathbf{b}_1||$. However, in practice this length cannot be increased indefinitely and therefore, error minimization is crucial in attitude determination problem using relative position observations. In this context,carrier phase measurement is crucial in devising a GPS-based attitude determination system.

1.2. Motivation and Impact

The importance of an attitude determination system in guidance, navigation, and control cannot be stressed enough. It is an integral part of a strapdown inertial navigation system and to a large extent determines the accuracy of the navigation solution. Many guidance algorithm consists of inner loop attitude control to perform guidance tasks such as waypoint tracking.

There is a recent growing interest on having a low-cost attitude determination system for land vehicle applications. Due to the limited space these vehicles operate in, they typically require high attitude accuracy to ensure safe operations. In this case, carrier phase signal is really needed to obtain sub-degree attitude accuracy. One of these applications is lane control. Data shows that in the recent years lane departure contributed to a major portion of fatal accidents in the United States. According to the National Highway Traffic Safety Administration (NHTSA) database, roadway departure accounts for almost 50% of the total fatalities in Minnesota from 2006 to 2010 [10]. Research on lane keeping control architecture [11, 12] has shown the importance of knowing the vehicle's heading accurately to prevent lane departure. One of the cheapest ways to generate heading estimate is by using magnetometer. However, its accuracy tends to be very low especially when operating in areas with significant magnetic disturbances. Although a navigation grade gyro can be used, its high unit cost make it prohibitive for automobile application. Therefore, a cheaper GPS-based attitude determination system capable of providing the same level of accuracy is very desirable.

In addition to direct applications of this GPS-based attitude determination system, the superior accuracy of this system also allows it to be a truth reference system for various research and development project. The ability to produce extremely accurate attitude allows calibration of other navigation sensors and algorithms [13]. Needless to say, there are a lot of other applications that can leverage such accuracy, especially when the cost is much lower than other systems that offer the same level of accuracy.

1.3. Prior Work and Contributions

There are many works that describes the use of carrier phase differential GPS (CPDGPS) method to design an attitude and heading reference system (AHRS). One of the earliest work was done by Cohen [14] at Stanford University. The more recent and successful work using a short baseline can be found on [12, 15, 16]. This system leverages on the short baseline to shrink the search space for integer ambiguity resolution. The flight test on a Beechcraft Queen Air demostrates the system's performance under real operating conditions. However, these successful works typically use high-performance GPS receivers that can preclude their use for low-cost applications. Additionally, these receivers usually outputs L2 carrier phase measurement, too.

In this work, we investigate the feasibility of using low-cost single-frequency original equipment manufacturer (OEM) receivers to determine the vehicle's heading. These receivers are generally more susceptible to cycle slip and signal interference and therefore could adversely affect the integer ambiguity resolution process. This investigation will give some insights on how to devise a carrier phase based AHRS under realistic condition. Stated differently, this work does not attempt to develop a new GPS-based AHRS algorithm but investigates the use of existing algorithms and methods for Intelligent Transportation System (ITS) applications at the University of Minnesota. In order to investigate the feasibility of having such an attitude system Additionally, although this work post-processed the collected data, the software is written in the way how a real-time software would process the data. Therefore, data collected on both the static and dynamic (road test) conditions would help determine which approach should be taken to devise such AHRS.

Finally, this work discusses the phase center variation error in the carrier phase measurement and efforts to calibrate it. Phase center error is a systematic error that results from the fact that the point where the signal's phase is measured do not coincide with the geometric center of the antenna and it can account up to 0.2 cm of an error in the carrier phase measurement [17]. Recent work such as [7] and [15] have shown that this error cannot be neglected when dealing with short baseline many land vehicle applications demand. However, due to time limitation, only preliminary calibration effort has been done. This report will comment on the challenges arising from the testing and recommend improvements for future work.

1.4. REPORT ORGANIZATION

This report is organized as follows. Chapter 2 gives an overview on the challenges encountered in real-time applications. More specifically, Section 2.1 compares and contrasts two integer ambiguity resolution techniques that can be applied for attitude determination problem. Namely, it discusses the integer search method (e.g. LAMBDA) and the attitude search space method [15]. Section 2.2 continues the discussion on systematic error with the phase center variation error. Finally, Chapter 3 presents the experiment setup and discusses the result. Chapter 4 closes the report with a summary and recommendations for future work.

CHAPTER 2

CARRIER PHASE SYSTEMATIC ERROR

The carrier phase measurement model in general can be modelled as follows.

$$\phi_k^i = r + \lambda \cdot \mathbf{N}_k^i + \beta - I_k^i + T_k^i + PCV_k^i + \epsilon \tag{2.1}$$

where the subscript k indicates the k-th receiver (e.g. base, rover 1 or rover 2) and the superscript i indicates the i-th satellite. Each term in Equation (2.1) has a physical significance. The first term, r_k^i, corresponds to the true geometric range from the geometric center of the k-th receiver to the i-th satellite. N is the integer ambiguity, and β is the combination of the receiver and the satellite clock bias error. I and T indicate the ionosphere and troposphere errors respectively. These atmospheric errors are caused by the change in the propagation speed as the signal travels through the media. The negative sign in front of the ionospheric term indicates the phase advance experienced by the carrier phase measurement, as opposed to the phase delay experienced by the pseudorange measurement. The sixth term, PCV, is the error associated with phase center variation and ϵ is unmodelled random error (e.g. thermal noise) that for practical purpose can be assumed to be white and Gaussian. Phase center variation term is rarely seen in GPS literature because it is negligible for most applications but those that require high accuracy.

The integer ambiguity, clock bias, ionosphere and troposphere erros, and the phase center variation constitute the systematic error of the carrier phase measurement. They are systematic because theoretically they can be modelled. The ionosphere and troposphere error are strictly a function of the signal propagation and, as will be discussed in more details later, phase center variation is a function of signal direction with respect to the antenna. With double difference technique[6], most of these systematic errors drop out of the final equation. For less than a kilometer of spacing between the two antennas, ionosphere and troposphere errors are cancelled and clock bias term drops out after double difference operation on the measurement. For most low cost antennas (less than $100), the phase center variation varies from unit to unit even despite being identical from the brand and model perspective. Therefore, double differencing the carrier phase measurements does not generally cancel this variation.

Integer ambiguity and phase center variation are subjects of this chapter. Section 2.1 discusses the integer ambiguity resolution and section 2.2 provides more detailed explanation on the phase center variation error. At this point, we note that phase center variation error is only up to approximately 0.2 cycles and can be neglected for integer ambiguity resolution purpose. It would be brought into the equations at the later stage to get a more accurate baseline vector solution.

2.1. INTEGER AMBIGUITY RESOLUTION

Integer ambiguity resolution is the process of solving for the ambiguity in Equation (1.1) while taking into account its integer nature. [7] provides a very detailed description of various integer ambiguity resolution techniques such as geometry-free technique, integer bootstrapping, and integer least square method. Most integer ambiguity resolution algorithm takes an initial estimate of the ambiguity and a covariance associated with the estimate as inputs. This initial estimate is usually not constrained as an integer (known as float estimate) and the algorithm will find a constrained solution that satisfies some optimality criteria. The float estimate can be obtained through several methods. One of the most popular method is by using a Kalman filter. Kalman filter is often preferred because it allows storing information from multiple epochs of observations and has covariance associated with the filter's estimate. This way, integer ambiguity resolution process is straightforward.

The algorithm that has received a lot of attention recently belongs to the class of integer least square method. In 1993, a group of researchers at TU Delft published the Least-square AMBguity Decorrelation Adjustment (LAMBDA) algorithm [18] that is reported to have superior efficiency compared to other search algorithm [19]. In this work, we investigate the feasibility of using LAMBDA for our application. In short, LAMBDA performs sequential conditional least square estimation preceded by a decorrelation of the ambiguities. [20] provides an explanation on the implementation aspect of LAMBDA. Discussion in Section 2.1.1 will give a brief overview of this method.

An older technique that can be used for attitude determination problem when the baseline is extremely short (less than 3 wavelength) is described in [15]. The method leverages the rotational nature of attitude determination problem and comes up with a float estimate by searching from a set of possible attitudes of the vehicle. As will be made clear in Section 2.1.2, the search efficiency decreases very quickly as the baseline length gets longer.

2.1.1. SEARCH IN THE INTEGER SPACE

LAMBDA resolves the integer ambiguity by minimizing a quadratic cost function of the weighted ambiguities error. The weighting matrix is chosen to be the inverse of the covariance matrix. The integer nature of the ambiguities is considered as a constraint to the optimization problem. Mathematically:

$$\check{N} = \arg\min_{N \in \mathbb{Z}} \left(N - \hat{N}\right) Q_{\hat{N}\hat{N}}^{-1} \left(N - \hat{N}\right) \tag{2.2}$$

where $\hat{N}$ and $Q_{\hat{N}\hat{N}}$ refer to the float estimate of the ambiguity and its associated covariance matrix.

Figure 2.1 illustrates the integer search space on a special case where there are only 2 integer ambiguities to be estimated. The black dot indicates the float estimate and every intersection of vertical and horizontal lines on the grid represents a valid integer pair. LAMBDA tries to find the correct integer pair indicated by the red dot on the figure. The size of the search space depends on the covariance matrix $Q_{\hat{N}\hat{N}}$.

A flow chart that depicts the integer ambiguity resolution process is shown on Figure 2.2. Key to LAMBDA's efficiency is the decorrelation process that occurs at the very beginning of the algorithm. After performing Cholesky decomposition on the covariance matrix to decorrelate the float estimates, an efficient integer search algorithm ranks all feasible candidates within the search space according to their respective cost index. Finally, the algorithm outputs the integer set with the lowest cost index. The optimality of this integer is claimed by comparing the cost index of the first two lowest integer

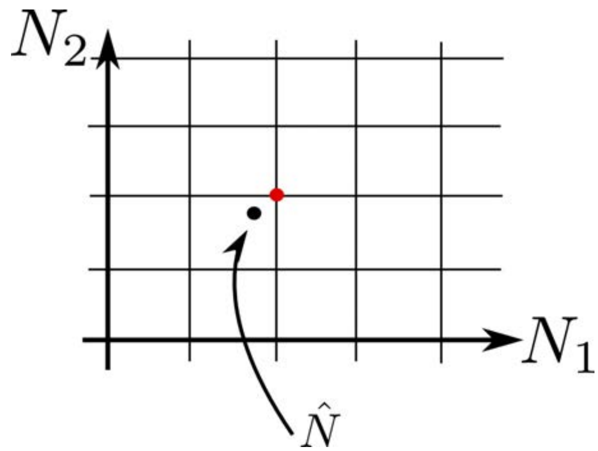

Figure 2.1: Integer search on 2-D space

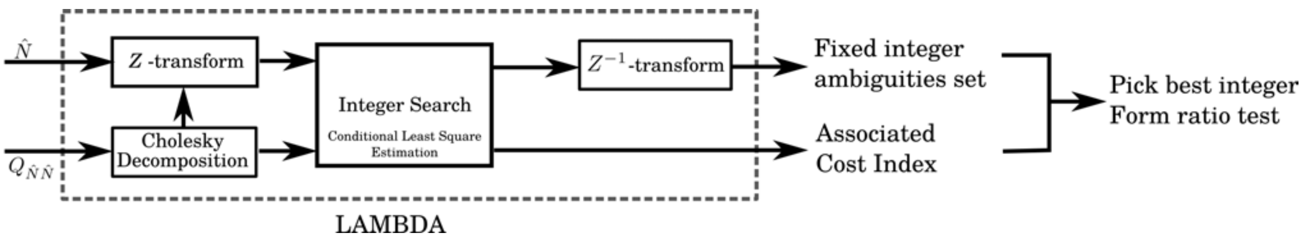

Figure 2.2: Schematic on LAMBDA

sets. In other words, if we have m possible integer sets and after ordering their cost index (CI) we have $0 \leq CI_{(1)} \leq CI_{(2)} \leq \cdots \leq CI_{(m-1)} \leq CI_{(m)}$, the ratio

$$r = \frac{CI_{(2)}}{CI_{(1)}} \geq 1 \tag{2.3}$$

can serve as a confidence indicator of the set picked. This ratio indicates the level of separation between the two best candidates in the cost index manifold. The ratio of 1 indicates indistinguishable separation between the best and the second best candidates, meanwhile larger value of r shows higher confidence that the candidates with the lowest cost index is indeed the correct integer set. This test is also known as the discrimination test.

Since the cost index is simply a sum of square of the error estimate, its distribution is Chi-squared if each integers are independent and identically distributed. When it is the case, the ratio test follows the F-distribution. Due to its simplicity, ratio test remains one very popular way to determine if the integer has been fixed correctly. Thresholding is usually selected as the fix criterion. Usually, we require the ratio to pass a pre-determined threshold and remain above the threshold for a fixed number of seconds before declaring a correct fix has been achieved. The selection of the value of this threshold, as shown in the next chapter, can be tricky although the widely accepted value for this threshold is 2 [21].

2.1.2. Search in the attitude space

With ultra short baseline (e.g. less than 1 wavelength), the integer search can be done very efficiently because there are only a few possibilities for the integer ambiguities. This can be understood by looking at the mathematical relationship in Equation (1.1) which after slight rearrangement is

$$\nabla\Delta\phi = (\mathbf{H}\cdot\mathbf{b}+\epsilon)+\lambda\cdot\nabla\Delta\mathbf{N} \tag{2.4}$$

If the baseline length is less than 1 wavelength, the terms in the paranthesis is on average less than 1 wavelength. Hence, if a round-off operation is performed on the entire right-hand side of the equation, the search space is constrained to only three possibilities for every satellite: 0 and ± 1. The number of combinations that needs to be checked is then

$$\text{\# of permutation} = 3^n \tag{2.5}$$

where n is the number of satellite tracked. For $n = 10$, this is 59049 combinations to check and anything more than a simple search algorithm is unnecessary.

As the baseline length increases, care must be exercised when using simple search method. Although the search space can be extended for longer baseline (e.g. for baseline less than 3 wavelengths, the search space is 0, ± 1, ± 2, ± 3), the space grows exponentially in size and the time it takes to search for the correct integer also grows very rapidly. However, for attitude determination problem, this problem can be worked around by leveraging the rotational nature of the problem. Since the baseline vector is known in the body frame and there is a unique rotation that transforms the vector in the body frame to the navigation frame, a search for an attitude close enough to the correct attitude can be performed so that the following residual error is less than a wavelength:

$$\mathbf{H}\cdot\mathbf{b}-\mathbf{H}\cdot\hat{C}_B^N\mathbf{b}^B < \lambda \tag{2.6}$$

where $\hat{C}_B^N$ indicates that the rotation matrix is a "guessed" attitude. Note that $C_B^N = \left(C_N^B\right)^T$. When Equation (2.6) is satisfied, a round-off operation on the residual can be performed and the correct integer can be found by searching in the set $\{0, \pm 1\}$. It has been shown in [15] that the correct integer, $\check{\mathbf{N}}$, can obtained by minimizing of the norm of the error:

$$\min_{\phi,\theta,\psi}\min_{\{0,\pm 1\}}\left\|\nabla\Delta\phi-\mathbf{H}\cdot\left(\hat{C}_N^B\right)^{-1}\mathbf{b}^B-\left(\tilde{\mathbf{N}}+\begin{bmatrix}\{0,\pm 1\}\\\{0,\pm 1\}\\\cdots\\\{0,\pm 1\}\end{bmatrix}\right)\right\| \tag{2.7}$$

over various feasible attitude ϕ, θ, and ψ, and over the possible combinations of $\{0, \pm 1\}$. $\tilde{\mathbf{N}}$ in Equation (2.7) is defined as

$$\tilde{\mathbf{N}} = \text{round}\left(\nabla\Delta\phi-\mathbf{H}\cdot\hat{C}_B^N\mathbf{b}^B\right) \tag{2.8}$$

A schematic that shows the steps in the algorithm is depicted on Figure 2.3.

Since roll, pitch and yaw angles are real quantities, the number of possible combinations is infinite. However, the attitude search does not need to be performed in this infinite set. The idea is that we need to get close enough to the correct attitude so that the correct attitude is included in the $\{0, \pm 1\}$ set. In other words, it is necessary that

$$\frac{1}{\lambda}\left\|\left(\hat{C}_N^B-C_N^B\right)\cdot\mathbf{b}^B\right\| < 1 \tag{2.9}$$

where C_N^B is the true attitude and $\hat{C}_N^B$ is again the "guessed" quantity. This equation allows us to find the maximum allowable increment in the attitude search. By using small angle approximation, it can

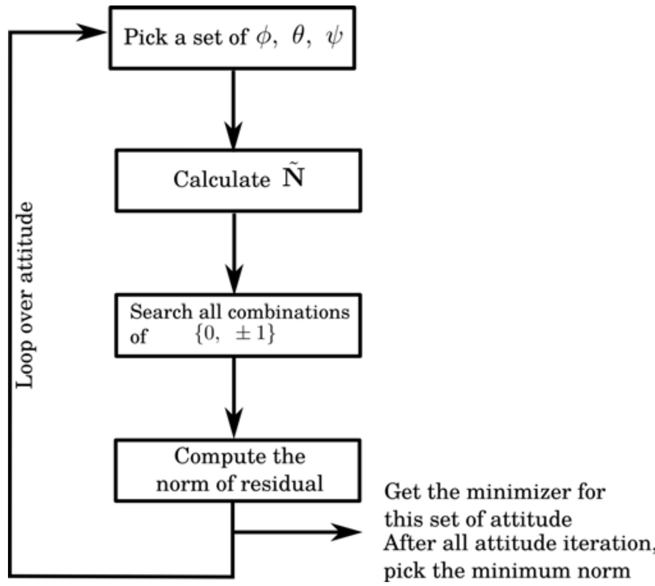

Figure 2.3: Flow diagram of search in attitude method

be shown that an increment of

$$\delta\phi,\ \delta\theta,\ \delta\psi < \frac{\lambda}{4\,||\mathbf{b}||} \tag{2.10}$$

ensures (2.9). For $||\mathbf{b}|| = 36\text{cm} < 3\lambda$, this translates to angular increment of 7.5 degrees in pitch, roll and yaw. The attitude search space can be further reduced if the pitch and roll motion are restricted to some range. For example, most land vehicle applications usually have restricted pitch and roll motion which allows the search in pitch and roll angles to be done only within some bounds. Additionally, Equation (2.10) also gives insights to how this search space increases as the baseline length gets longer. Hence, this method is only effective when short baseline length (typically less than 3 wavelength) is used.

2.2. Phase Center Variation

One of the significant factor that contribute to attitude error is phase center variation (PCV) error [7, 12, 15]. Phase center refers to a point where the carrier phase measurement is made. Due to antenna design, the phase center of the antenna might not coincide with the geometric center of the antenna. And since the relative position vector given by carrier phase measurement is the relative position between two antenna phase centers rather than the geometric centers, this results in position error and thus attitude error. Figure 2.4 illustrates the difference between the phase center and the geometric center of the antenna.

Measuring the location of the phase center of the antenna is a very delicate task. Not only because it cannot be measured using a measurement device such as a measuring tape, but also because its location is a function of signal direction, power intensity, and frequency of the signal [7]. Following the model developed by researchers in National Geodetic Survey (NGS) [17], the antenna's phase center is a sum of two components: the mean phase center offset and the PCV.

$$\vec{r}_{pc} = \vec{r}_m + \vec{r}_{pcv} \tag{2.11}$$

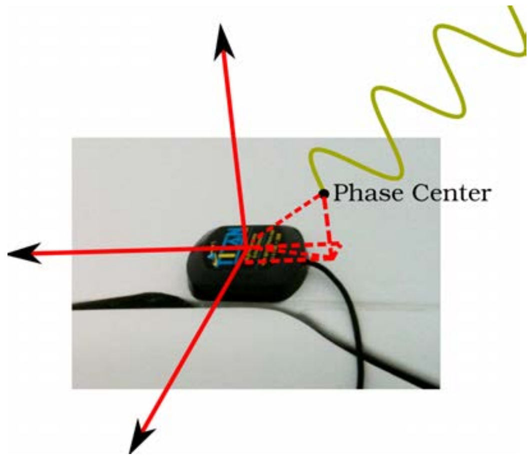

Figure 2.4: Illustration of phase center variation with respect to antenna

For a given installation (and hence nominal signal power configuration) the mean phase center offset is modelled as a constant bias with respect to the nominal baseline vector. The remaining variation from incoming signal direction (azimuth and elevation) is captured by PCV. Due to the symmetrical design of the antenna, the variation with respect to azimuth is often negligible[7]. A 4th-order model with respect to the signal elevation is adopted. Since this elevation angle varies from one satellite to another, each carrier phase measurement is subject to a different variation. Thus, it is easier to model this error for individual satellite's carrier phase measurement rather than in the position domain ($\vec{r}_{pcv}$). In the unit of cycle, it can be mathematically written as

$$C_k^i = C_0 + C_1 E + C_2 E^2 + C_3 E^3 + C_4 E^4 \qquad (2.12)$$

where E is the elevation angle of the incoming signal direction.

Using this model, one only needs to vary the signal direction and measure the carrier phase measured on the antenna itself. There are two main methods to vary the signal direction. In the first method, the whole system is put in an anechoic chamber where artificial GPS signal can be simulated and the signal direction can be varied. This method is known as the laboratory method. The second method uses the satellites' motion to vary the incoming signal direction. This is an experimental method that involves a very long data collection period in order to get the complete phase center map. Moreover, the process needs to be done on a relatively open environment in order to avoid calibrating multipath errors onto the coefficients C_0, C_1, C_2, C_3, and C_4. Details on the calibration process can be found on [7].

CHAPTER 3

RESULT AND DISCUSSION

To collect experimental data, a set of three GPS receivers and antenna pairs is assembled. Hemisphere Crescent OEM board is chosen for this work due to its low cost and simple interfacing. Titan III is the antenna of choice. It is a high performance active GPS antenna made by GPS Outfitters[1]. Each receiver is configured to send binary data at 1 Hz containing the position and velocity information, the pseudorange measurements, and the carrier phase measurements. The data stream from each receiver is logged by a serial data logger that saves the data into a micro SD card. The unit is battery powered, and a 3.3 V regulator maintains constant voltage to power the unit. Table 3.1 lists all the hardwares and the unit price for each component. The picture on the left of Figure 3.1 shows the final assembly of the unit.

The software to post-process the data is written in MATLAB and has been written to take the raw data and process it as a real-time system would process it. The majority of the software deals with the bookkeeping required to manage the huge amount of information sent by the receiver. The core algorithm then processes the raw measurements into position information. It consists of a Kalman Filter that calculates the float estimate and an integer ambiguity resolution algorithm that fixes the integer ambiguity and outputs the fixed estimate of the baseline vector.

In Section 3.1, the integer ambiguity resolution performance in realistic and real-time environment is discussed. The analysis is done based on data collected on several road tests to show a realistic performance of this attitude system. Finally, Section 3.2 will elaborate the challanges encountered in calibrating the phase center variation.

Table 3.1: List of major hardware component and price

Component	**Unit Price (US$)**
Hemisphere Crescent OEM board	280
Titan III antenna	70
Serial data logger	25
Voltage converter and power regulator	18
Micro SD Card	5

[1]http://www.gpsoutfitters.com/Titan3.html

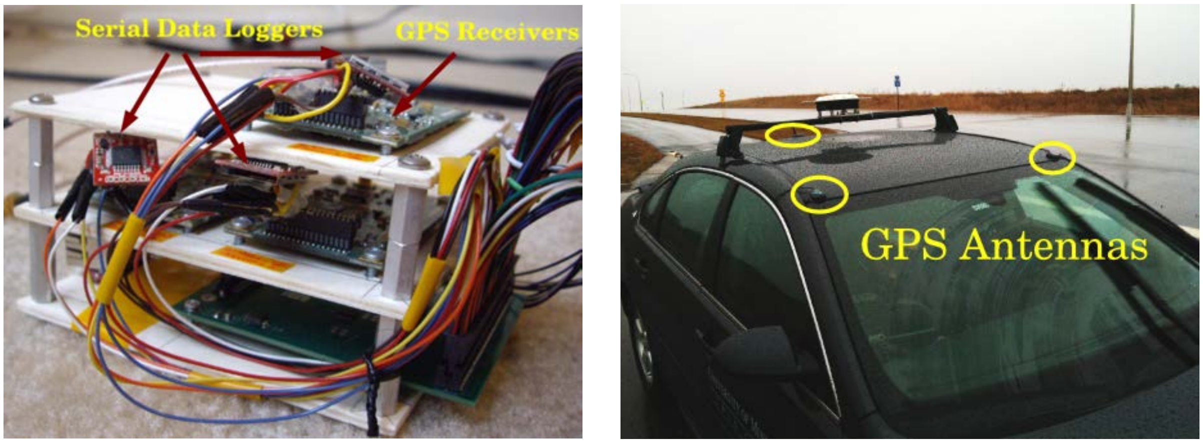

Figure 3.1: Hardware setup showing 3 GPS receivers each with its independent data logger (left) and the arrangement of the GPS antenna during car test (right)

3.1. Performance and Feasibility Analysis

A series of static and dynamic data collection was done to examine the performance of two integer ambiguity algorithms. The first method investigated is LAMBDA and the second one is the attitude search method described in Section 2.1.2. The right picture on Figure 3.1 depicts the three Titan III antennas mounted on the test vehicle. The GPS receiver setup shown in Figure 3.1 was located inside the car. One antenna was mounted at the rear and played the role of a base receiver (B). Two other antennas were mounted on the front left and right. They served as the rover 1 (R1) and rover 2 (R2) receivers.

Figure 3.2 and 3.3 show typical results obtained from LAMBDA on a static test. The vector from base to rover 1 is denoted as BR1 and the vector from base to rover 2 is denoted BR2. Both baselines are 1.46 m long. The reason this baseline configuration is chosen is mainly for accuracy. Since this is the maximum length we can get for BR1 and BR2 (given the car's configuration), this will give us the minimum attitude error. However, since this length is approximately 770% of the wavelength, the attitude search method cannot be applied in this particular test.

In both figures, the plot on the left hand side shows the baseline vector components and its respective length (1.46 m). Each components (North, East, Down, and length) is plotted in different colors described in the legend. The solid lines and dashed lines indicate two different estimates. The solid lines are the float estimate from the Kalman Filter that is continuously fed into LAMBDA to get a fixed baseline vector. The fixed baseline vector components are plotted as dashed lines. The true baseline vector for BR1 is

$$\mathbf{b}_{BR1}^{N} = [-1.37 \qquad -0.50 \qquad 0.01]^{T}\ \mathrm{m} \tag{3.1}$$

and for BR2 is

$$\mathbf{b}_{BR2}^{N} = [-1.37 \qquad 0.50 \qquad 0.01]^{T}\ \mathrm{m} \tag{3.2}$$

Both true BR1 and BR2 vector components are known *a priori* (static test). The plots on the right hand side on both Figure 3.2 and 3.3 show the ratios as described in Section 2.1. In a real-time setting, ratio test is the easiest indicator of the fix quality. High ratio indicates high confidence in the integer obtained from LAMBDA.

In Figure 3.2, the dashed lines on the left plot shows the correct baseline vector (Equation (3.1)) components right after the first epoch observation. Correct baseline vector indicates correct integer fix. This almost immediate correct integer resolution happened occasionally in several of our tests and it is clear that this very good performance really depends on the signal quality. Better signal quality typically results in faster resolution time. Nevertheless, the ratio test as plotted on the right hand side might not reflect the same situation. There are many ways to declare a fix. If the threshold of 2 is chosen and if we require the ratio test to remain above the threshold value for several number of seconds, then in this particular data set the integer ambiguity resolution takes approximately 4 minutes to reach the required level of confidence. The float estimates (solid lines) also converge to their correct values (± 5 cm) after 200 seconds.

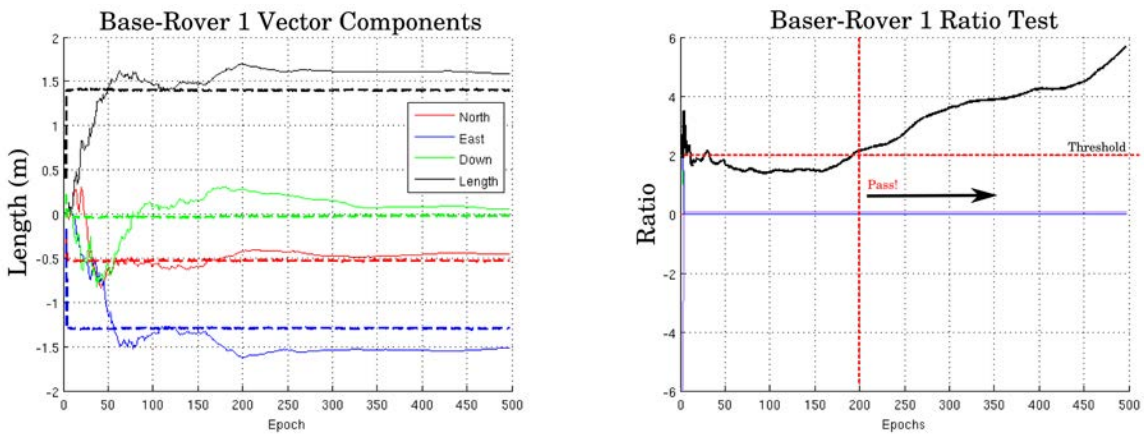

Figure 3.2: Base-Rover1 baseline vector and the ratio test, $||\mathbf{b}_{BR1}||$ = 1.46 m

The second baseline vector, BR2, is depicted in Figure 3.3. Opposite to the result obtained from the first baseline, several discrete jumps in the fixed estimate (dashed line) occured. These jumps are results of integer switching that occurs when it no longer minimizes the cost function in LAMBDA. This is also clear from the ratio test shown on the right hand side of Figure 3.3. The discrete jumps occur when the ratio test is equal to 1, an indication that what was previously the second best set of integer is now the set of integer that minimizes the cost function. By comparing the fixed estimate with the true baseline vector (Equation (3.2)), it took the algorithm 300 seconds to get the correct integer. The float solutions (solid lines) on the other hand has converged to their correct values (± 5 cm) after 200 seconds; similar to BR1. In real-time situation where we don't have access to the true baseline vector, ratio test remains as the device to declare integer fix. Unlike in BR2, the test could not confirm the integer resolution in 500 seconds although there is a clear trend that ratio test will continue to grow beyond the data available.

It is clear from this static test that there is a challenge in fixing integer ambiguity in real-time using single-frequency low-cost GPS receiver. The biggest challenge lies in the difficulty to declare with confidence that the baseline vector. This problem is amplified with cycle slips that occur as the tracking loop stops integrating the carrier phase measurements internally and thus resets the integer ambiguity in the channel that slips. Cycle slips occur as the receiver experiences large accelerations or when the carrier noise is too large [12]. Cycle slips slow down the integer ambiguity resolution process significantly because the channel that slips needs to be re-initialized.

The system is tested under dynamic condition to investigate the challenge of fixing the integer on-the-go. Although the results from static tests are not very encouraging, but the goal of this test is also to investigate the usefulness of the attitude search algorithm. In this test, only a single baseline is

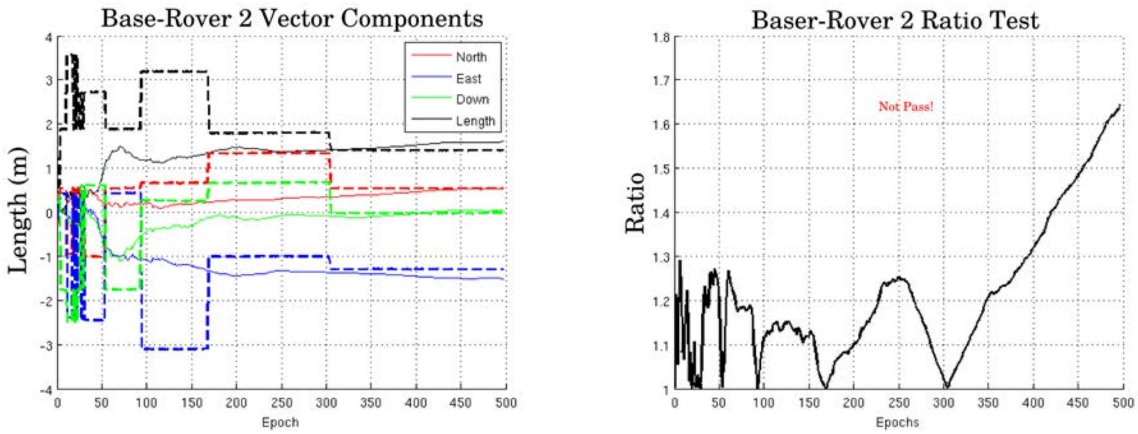

Figure 3.3: Base-Rover1 baseline vector and the ratio test, $||\mathbf{b}_{BR2}||$ = 1.46 m

examined and the length is shortened to 36 cm (< 3 wavelength) The pictures on the top of Figure 3.5 show the experimental set up and the test scenario.

Figure 3.4 compares the estimates using two ambiguity resolution techniques. The picture on the left plots the result using LAMBDA and the plot on the right-hand side depicts the baseline vector obtained by using the attitude search algorithm. Since the baseline vector is aligned with the car's axis, it is not possible to estimate roll angle with this setup. The attitude search is performed on the yaw and pitch axes. The complete 360° yaw axis is discretized at 5° increment. Since the car only experiences limited pitch motion, the pitch axis is searched from -10° to 10° at 5° increment. In the data set plotted in Figure 3.4 the car starts moving at t = 12 seconds. As the plot on the left hand side show, LAMBDA is unable to resolve the integer ambiguity correctly and the baseline vector estimate is incorrect. On the other hand, the attitude search method results in a very convincing results.

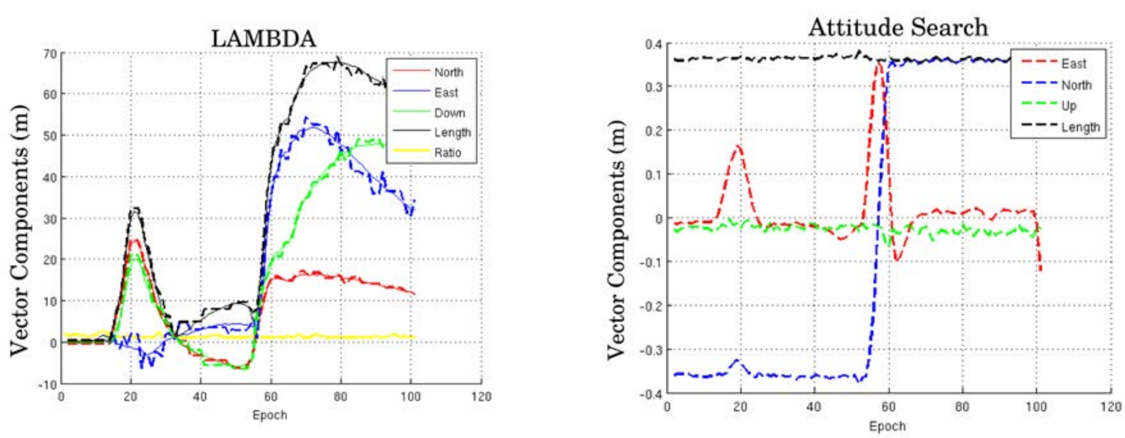

Figure 3.4: Comparison between integer search and attitude search method on a short baseline, $||\mathbf{b}||$ = 0.36 m)

Figure 3.5 compares the attitude derived from the baseline vector components (plotted in red) with the track angle derived from the car's velocity (plotted in blue). The track angle derived from the velocity measurement is unavailable when the car is not moving (first 200 seconds and last 100 seconds). However, when the car is moving, both heading angles are in great agreement.

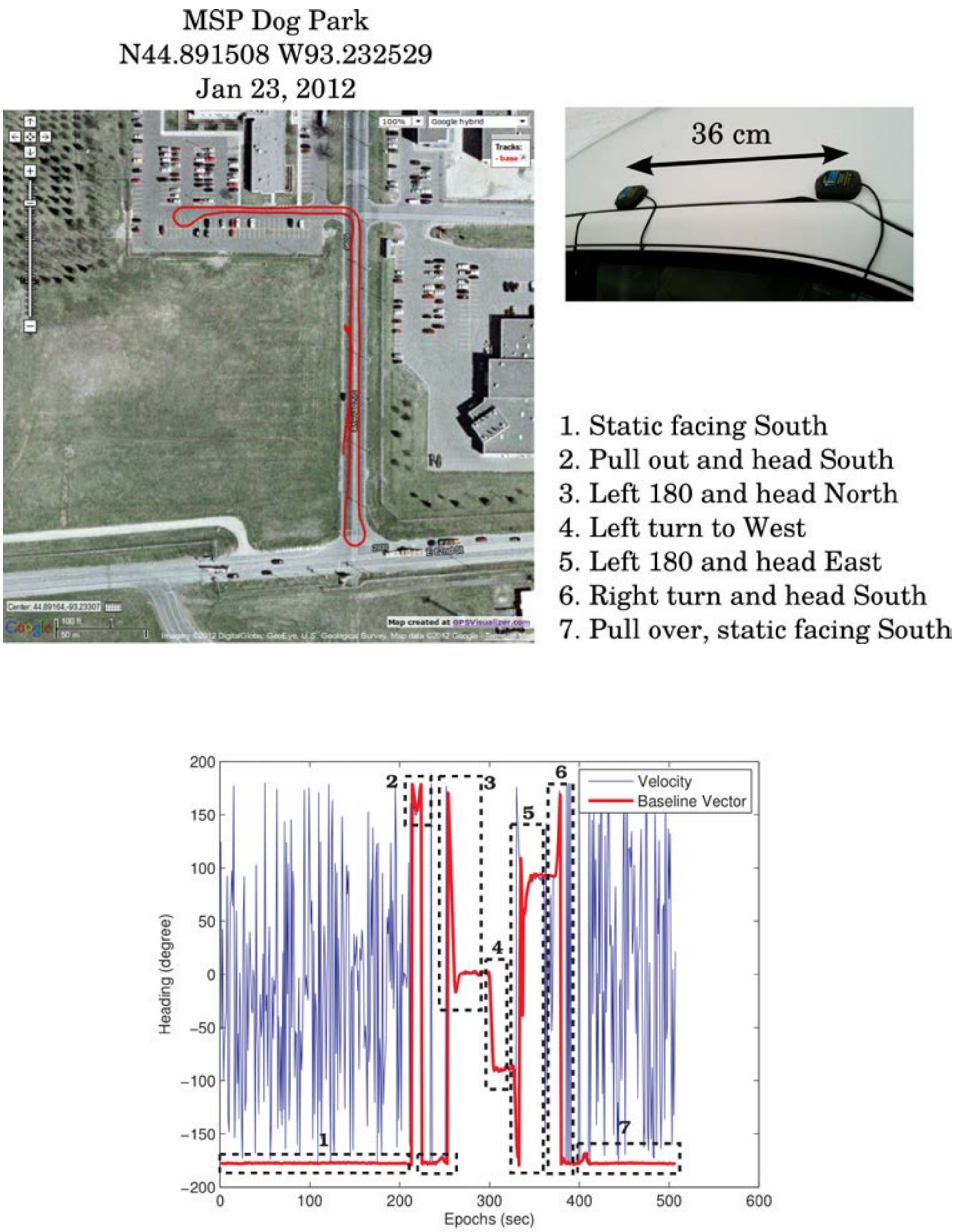

Figure 3.5: Dynamic trajectory (left) and heading (right) plot of a single baseline heading determination system

3.2. Phase Center Calibration

NGS website [22] lists calibration of most of high-end antenna available in the market using the experimental method. These antennas are expensive (more than $1,000) and are mostly big which precludes their use on short baseline applications. Hence, despite their readily available calibration data, the antennas listed in NGS website are not suitable candidates for our application. Titan III is not listed on the website and in this project we try to mimic the experimental setup from NGS to calibrate a set of three Titan III's. Figure 3.6 depicts the antenna configuration setup for the calibration. The data collection was done at the Minnesota Valley Transit Authority (MVTA) garage rooftop in Eagan, MN.

Figure 3.6: Phase center calibration test setup

The data collection lasted for 24 hours and each receiver logged their respective binary output to the data logger. The data is post-processed to obtain the coefficients C_0 to C_4 in Equation (2.12). Integer ambiguity is fixed by using the true baseline vector and rounding off the float estimate. The remaining residuals would be due to phase center variation.

The only difference between this experimental setup and the NGS setup is with regards to the clock that drives the GPS receiver. In NGS setup, there is one external oscillator that drives all receivers so that they run on a single clock. Therefore, single differencing (between receivers) removes not only atmospheric errors but also the clock bias error. In our setup, each GPS receiver runs on their own built-in clock. Hence, each has its own clock bias that needs to be removed through double differencing the measurements.

Double differencing effectively increases the noise level in the carrier phase measurement. However, because of proprietary issue and time constraint, we were unable to tie the clock of our GPS receivers to avoid double differencing. As it turns out, the noise level after double differencing is high enough to affect the ability to capture the phase center variation in our collected data. Limited time has not allowed us to make modification on our hardware setup and collect more data. Nevertheless, this preliminary analysis should serve well as a starting point for future work in this area.

CHAPTER 4

SUMMARY AND CONCLUSION

In this report, attitude determination using relative position vector obtained from GPS carrier phase measurement is discussed. Two main issues pertaining to GPS-based attitude determination system namely the integer ambiguity resolution and phase center variation are highlighted. Integer ambiguity resolution techniques and validation are examined in the context of real-time application.

Two methods for ambiguity resolution techniques namely LAMBDA and attitude search method are examined for their respective feasibility for ITS applications. Careful examinations on data obtained from a series of road tests show that LAMBDA is not the most effective method to resolve integer ambiguity in real-time environment. One of the biggest drawbacks of the algorithm is the lack of validation method to assure the integrity of the resulting integer estimates. However, AHRS applications that allow the use of short baseline ($< 3\ \lambda$) can benefit from the use of attitude search method to fix the integer ambiguity. Static and dynamic tests carried out in this work show its superior performance in real-time setting.

The attitude search space method requires a short baseline vector to meet the maximum allowable computation time in real-time operation. In this case, phase center variation error becomes a significant error component that needs to be calibrated to achieve sub-degree attitude accuracy. Some preliminary work has been done to calibrate the phase center variation error. Factors that affect the success of this calibration were discussed. This analysis would help direct future work.

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